Graphs of Equations in Two Variables; Intercepts; Symmetry

The ***solutions*** of an equation with two variables are pairs of numbers that make the equation true when they are substituted in for the variables.

Graph of an Equation: A drawing that represents all the solutions of an equation. Any ordered pair on the graph is a solution to the equation.

Example: Determine whether the points $(0,0)$, $(1,1)$, and $(1,-1)$ are on the graph of $y = x^3 - 2\sqrt{x}$.

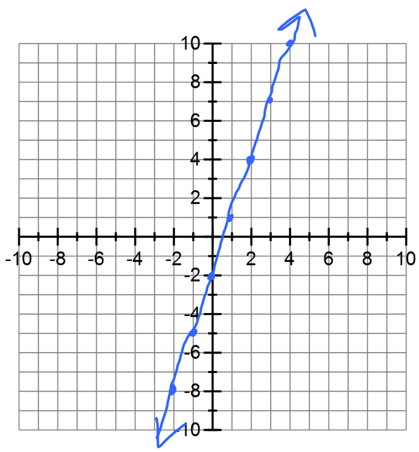
$(0,0)$: $0 \stackrel{?}{=} 0^3 - 2\sqrt{0}$
$0 = 0$ ✓
$\boxed{\text{yes}}$

$(1,1)$ $1 \stackrel{?}{=} 1^3 - 2\sqrt{1}$
$1 \stackrel{?}{=} 1-2$
$1 \neq -1$
$\boxed{\text{no}}$

$(1,-1)$: $-1 \stackrel{?}{=} 1^3 - 2\sqrt{1}$
$-1 \stackrel{?}{=} 1-2$
$-1 = -1$ ✓
$\boxed{\text{yes}}$

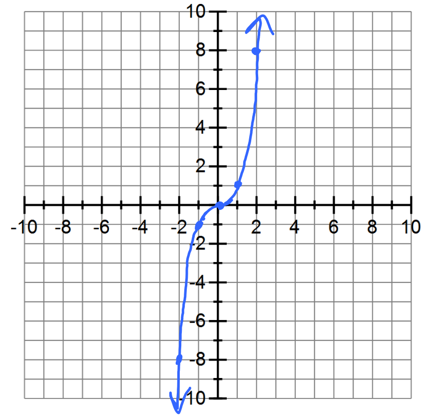
Examples: Graph the following equations by plotting points.

a) $y = 3x - 2$



x	$y = 3x-2$
-2	$3(-2)-2 = -8$
-1	$3(-1)-2 = -5$
0	$3(0)-2 = -2$
1	$3(1)-2 = 1$
2	$3(2)-2 = 4$
3	$3(3)-2 = 7$

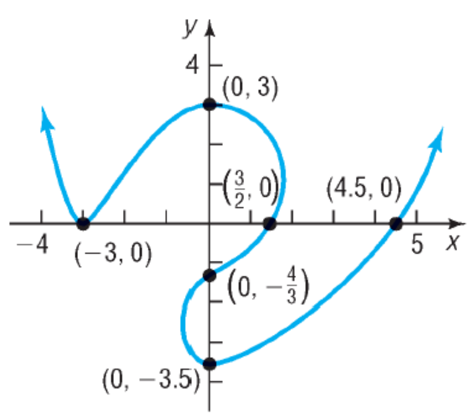
b) $y = x^3$



x	$y = x^3$
-2	$(-2)^3 = -8$
-1	$(-1)^3 = -1$
0	$0^3 = 0$
1	$1^3 = 1$
2	$2^3 = 8$

Intercepts: The points at which a graph crosses or touches the coordinate axes. The x-coordinates of the points where the graph crosses the x-axis are called ***x-intercepts***. The y-coordinates of the points where the graph crosses the y-axis are called ***y-intercepts***.

Example: Find all the intercepts of the graph.



x-intercepts: $-3, \frac{3}{2}, 4.5$
y-intercepts: $-3.5, -\frac{4}{3}, \&\ 3$

Intercepts: $(-3,0), (\frac{3}{2},0), (4.5,0),$
$(0,-3.5), (0,-\frac{4}{3}), (0,3)$

Notice that at the x-intercepts, $y = 0$, and at the y-intercepts, $x = 0$.

Finding Intercepts from an Equation

1. To find the x-intercept(s), let $y = 0$ and solve for x.
2. To find the y-intercept(s), let $x = 0$ and solve for y.

Example: Find the x-intercept(s) and y-intercept(s) of the graphs.

a) $y = x^2 - 6x + 16$

y-int: $y = 0^2 - 6(0) + 16$
$y = 16$
$\boxed{(0,16)}$

x-ints: $0 = x^2 - 6x + 16$
Quadratic formula
$x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(1)(16)}}{2(1)}$
$x = \dfrac{6 \pm \sqrt{-28}}{2}$
imaginary
$\boxed{\text{no x-intercepts}}$

b) $4x^2 + 9y^2 = 36$

x-ints: $4x^2 + 9(0)^2 = 36$
$4x^2 = 36$
$x^2 = 9$
$x = \pm 3$
$\boxed{(3,0) \text{ \& } (-3,0)}$

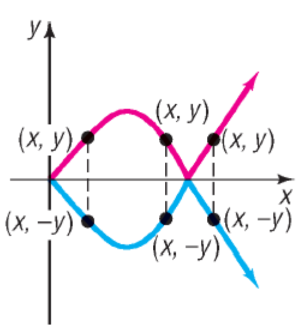
y-ints: $4(0)^2 + 9y^2 = 36$
$9y^2 = 36$
$y^2 = 4$
$y = \pm 2$
$\boxed{(0,2) \text{ \& } (0,-2)}$

Symmetry

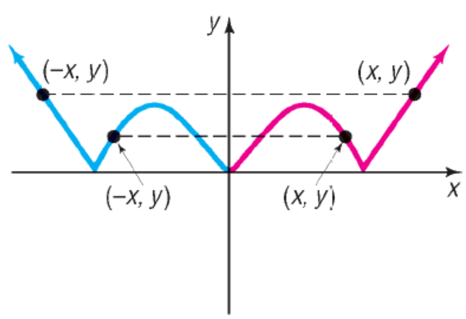
1. A graph is said to be ***symmetric with respect to the x-axis*** if, for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph. Test by replacing y by $-y$ in the equation and simplifying. If an equivalent equation results, the graph of the equation is symmetric with respect to the x-axis.

2. A graph is said to be ***symmetric with respect to the y-axis*** if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph. Test by replacing x by $-x$ in the equation and simplifying. If an equivalent equation results, the graph of the equation is symmetric with respect to the x-axis.

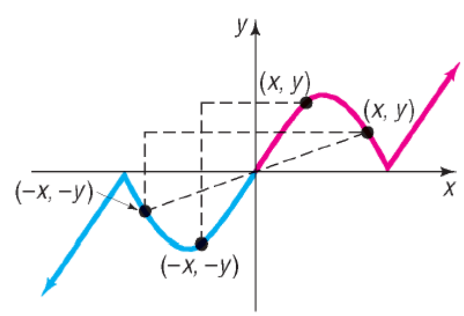
3. A graph is said to be ***symmetric with respect to the origin*** if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph. Test by replacing x by $-x$ and y by $-y$ in the equation and simplifying. If an equivalent equation results, the graph of the equation is symmetric with respect to the origin.



Symmetry with respect to the x-axis



Symmetry with respect to the y-axis



Symmetry with respect to the origin

Examples: Test each equation for symmetry.

a) $y = \dfrac{x^2 - 4}{2x}$

x-axis: Replace y with -y
$-y = \dfrac{x^2-4}{2x}$
$y = -\left(\dfrac{x^2-4}{2x}\right)$ no

y-axis: Replace x with -x
$y = \dfrac{(-x)^2 - 4}{2(-x)}$
$y = \dfrac{x^2-4}{-2x}$ no

origin: Replace x with -x & y with -y
$-y = \dfrac{(-x)^2-4}{2(-x)}$
$-1(-y) = -1\left(\dfrac{x^2-4}{-2x}\right)$
$y = \dfrac{x^2-4}{2x}$ yes

$\boxed{\text{Symmetric with respect to the origin}}$

b) $y = \sqrt[5]{x}$

x-axis: $-y = \sqrt[5]{x}$
$y = -\sqrt[5]{x}$ no

y-axis: $y = \sqrt[5]{-x}$
$y = -\sqrt[5]{x}$ no

origin: $-y = \sqrt[5]{-x}$
$-y = -\sqrt[5]{x}$
$y = \sqrt[5]{x}$ yes

$\boxed{\text{Symmetric with respect to the origin.}}$

c) $4x + y^2 = 4$

x-axis: $4x + (-y)^2 = 4$
$4x + y^2 = 4$ yes

y-axis: $4(-x) + y^2 = 4$
$-4x + y^2 = 4$ no

origin: $4(-x) + (-y)^2 = 4$
$-4x + y^2 = 4$ no

Symmetric with respect to the x-axis.

d) $9x^2 + 4y^2 = 36$

x-axis: $9x^2 + 4(-y)^2 = 36$
$9x^2 + 4y^2 = 36$ yes

y-axis: $9(-x)^2 + 4y^2 = 36$
$9x^2 + 4y^2 = 36$ yes

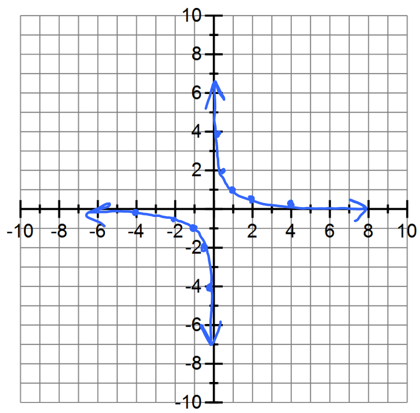
origin: $9(-x)^2 + 4(-y)^2 = 36$
$9x^2 + 4y^2 = 36$ yes

Symmetric with respect to the x-axis, y-axis, and origin.

Graphing Key Equations

1. Find x-intercepts and y-intercepts.
2. Check for symmetry with respect to the x-axis, the y-axis, and the origin.
3. Create a T-table based on the results of step 1 & 2.

a) $y = 1/x$



x	$y = 1/x$
0	1/0 is undefined
1/4	4
1/2	2
1	1
2	1/2
4	1/4
-1/4	-4
-1/2	-2
-1	-1
-2	-1/2
-4	-1/4

x-int: $0 = \frac{1}{x}$
$0 = 1$
$\boxed{\text{no x-int}}$

y-int: $y = \frac{1}{0}$
$\boxed{\text{no y-int}}$

Symmetry:

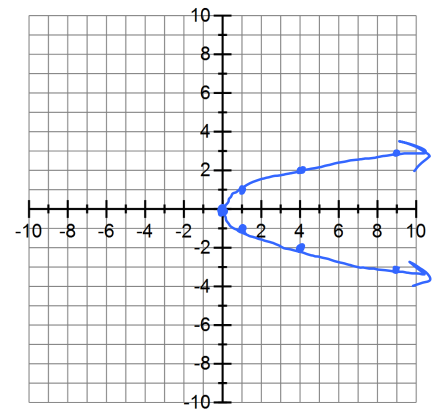
x-axis: $-y = 1/x$
$y = -1/x$ no

y-axis: $y = \frac{1}{-x}$ no

$\boxed{\text{origin}}$: $-y = \frac{1}{-x}$
$y = \frac{1}{x}$ yes

If (x,y) is on the graph, so is $(-x,-y)$

b) $x = y^2$



$x = y^2$	y ← pick these
4	-2
1	-1
0	0
1	1
4	2

x-int: $x = 0^2$
$x = 0$

y-int: $0 = y^2$
$y = 0$

Symmetry:

$\boxed{\text{x-axis}}$: $x = (-y)^2$
$x = y^2$ yes

y-axis: $-x = y^2$
$x = -y^2$ no

origin: $-x = (-y)^2$
$-x = y^2$
$x = -y^2$ no

If (x,y) is on the graph, is $(x,-y)$